Lecture 15. Rate of return on an investment

FOR 2022. Financial Analysis for Natural Resources.

School of Forest Resources

Why is determining a rate of return so important?

- A simple decision criteria
- An investment that returns 8% interest is better than one that returns 6%, all other things being equal (*ceterus parabus*)
- The general public readily understands rates of return
- No need to determine some time preference for resources or the best alternative rate of return

Earning rate

- Simple situation
- Initial cost (V_0) and some ending return (V_n).
  - No intermediate cash flows
- Earning rate equation is:

$$i = \left[\sqrt[n]{\frac{V_n}{V_0}} - 1 \right] 100$$

Where,

n = number of discount periods

And earning rate (i) is expressed as a percentage

Determining earning rate

- You're great aunt Tilly gives you a $500 U.S. Series EE savings bond that she bought when you were born, 22 years ago. You take it to the bank and find out that it is worth $811.88.
- What was the annual rate of return on this bond (assuming Aunt Tilly bought it for ½ face value)?

$$i = \left[\sqrt[n]{\frac{V_n}{V_0}} - 1 \right] 100$$

$$i = \left[\sqrt[22]{\frac{\$811.88}{\$250}} - 1 \right] 100 = 5.5\%$$

Determining internal rate of return (IRR)

- Used when a project has intermediate cash flows.
- Assumes that all intermediate cash flows can be reinvested at the internal rate of return.
  - Usually the case with many similar projects being started each year.
- IRR defined as the interest rate that causes net present value (NPV) to equal zero
- Determined by inspection!
  - No formula, you just keep trying different interest rates until NPV = 0!

Determining IRR

Year	Cash Flow	Present value at:		
		4%		
0	($250)	($250)		
8	(90)	(66)		
20	$500	228		
30	$1926	594		
NPV =		506		

Used simple present value equation: $V_0 = V_n(1+i)^{-n}$

$$V_0 = (\$250(1.04)^{-0} = (\$250)$$
$$V_0 = (\$90)(1.04)^{-8} = (\$66)$$
$$V_0 = \$500(1.04)^{-20} = \$228$$
$$V_0 = \$1926(1.04)^{-30} = \$594$$

Since NPV > 0 at 4%, we'll try 12%...

Determining IRR

Year	Cash Flow	Present value at:		
		4%	12%	
0	($250)	($250)	($250)	
8	(90)	(66)	(36)	
20	$500	228	52	
30	$1926	594	64	
NPV =		506	(170)	

Used simple present value equation: $V_0 = V_n(1+i)^{-n}$

$$V_0 = \$500(1.12)^{-20} = \$52$$

Since NPV < 0 at 12%, we'll try 8%...

Determining IRR

Year	Cash Flow	Present value at:		
		4%	12%	8%
0	($250)	($250)	($250)	($250)
8	(90)	(66)	(36)	(49)
20	$500	228	52	107
30	$1926	594	64	191
NPV =		506	(170)	0

Used simple present value equation: $V_0 = V_n(1+i)^{-n}$

$$V_0 = \$500(1.08)^{-20} = \$107$$

NPV = 0 at 8%, so IRR = 8%

Composite rate of return or realizable rate of return (RRR)

- Necessary when reinvestment rate is not same as internal rate of return
  - Intermediate cash flows (costs and returns) are discounted at some actual reinvestment rate.
- Let's use our previous example. Let's say that we could reinvest all possible intermediate cash flows at 5%...

At 5% reinvestment rate…

We can calculate the present value of all costs at 5%, and we can also determine the future amount of cash we will have on hand when the project ends by determining the future value of all returns!

Year	Cash Flow	Present value at 5%	Future value at 5%
0	($250)	$\$250(1.05)^{-0} = \$250$	
8	(90)	$\$90\ (1.05)^{-8} = \$61$	
20	$500		$\$500(1.05)^{(30-20)} = \$814$
30	$1926		$\$1926(1.05)^{(30-30)} = \$1926$
Totals		$311	$2740

Now we can use the earning rate equation to find RRR:

$$i = \left[\sqrt[30]{\frac{\$2740}{\$311}} - 1 \right] 100 = 7.5\%$$

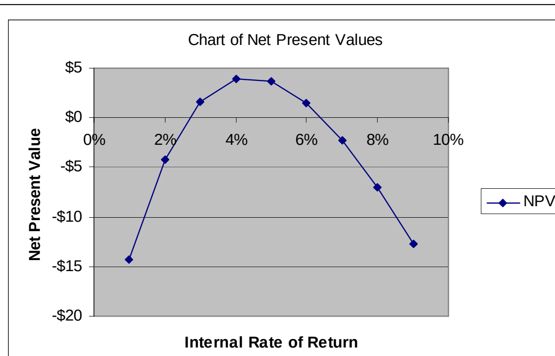
A "generalized" formula for RRR

$$RRR = \left[\sqrt[n]{\frac{\sum_{t=0}^{n} R_t (1+i)^{n-t}}{\sum_{t=0}^{n} C_t (1+i)^{-t}}} - 1 \right] 100$$

Where: R_t = some return in year t
C_t = some cost in year t
n = total number of discount periods
i = reinvestment rate

Projects with alternating positive and negative cash flows can have multiple IRRs!

Year	Cash Flow	1%	2%	3%	4%	5%	6%	7%	8%	9%
0	-$250	-$250	-$250	-$250	-$250	-$250	-$250	-$250	-$250	-$250
5	$420	$400	$380	$362	$345	$329	$314	$299	$286	$273
20	-$200	-$164	-$135	-$111	-$91	-$75	-$62	-$52	-$43	-$36
	NPV =	-$14	-$4	$2	$4	$4	$1	-$2	-$7	-$13



Test your knowledge

- Part A: Using the following cash flow table, determine the IRR for the project
- Part B: If your reinvestment rate is 6%, what is the projects realizable rate of return?

Year(s)	Cash Flow
0	(141)
1-25	(4)
5	(75)
15	$200
25	$1500

Internal Rate of Return

Year(s)	Cash Flow	2%	4%	6%	8%	9%	12%
0	-$141	-$141	-$141	-$141	-$141	-$141	-$141
1-25	-$4	-$78	-$62	-$51	-$43	-$39	-$31
5	-$75	-$68	-$62	-$56	-$51	-$49	-$43
15	$200	$149	$111	$83	$63	$55	$37
25	$1,500	$914	$563	$349	$219	$174	$88
	NPV =	$776	$409	$185	$47	$0	-$90

Our internal rate of return is 9%!

Formula for present value of terminating series of annual payments

$$V_0 = (\$4)\frac{1-(1.06)^{-25}}{0.06} = (\$51)$$

Realizable Rate of Return

At 6%, the present value of all our costs can be determined as:

$$V_0 = \$141(1.06)^{-0} + \$4\frac{1-(1.06)^{-25}}{0.06} + \$75(1.06)^{-15} = \$141 + \$51 + \$56 = \$248$$

And at 6%, the future value of all returns are:

$$V_{25} = \$200(1.06)^{25-15} + \$1{,}500(1.06)^{25-25} = \$358 + \$1{,}500 = \$1858$$

So, realizable rate of return is:

$$i = \left[\sqrt[25]{\frac{\$1858}{\$248}} - 1\right]100 = 8.4\%$$

Next lecture…

Financial narratives and net present value revisited….