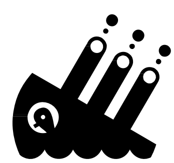
Lecture 9. Sinking fund and capital recovery equations.

FOR 2022. Financial Analysis for Natural Resources.

School of Forest Resources

What is a sinking fund?



- You are depositing money into some form of account or investment with the goal of a set amount of money at some future time.
  - You know future amount
  - You know number of compounding periods
  - You know the interest rate
  - You must solve for the annual amount
- Example: You want to have $10,000 saved in 4 years. Your bank pays you an annual interest rate of 3% per year, compounded monthly. How much must you deposit each month?
- A variation on the future value of a terminating series of annual payments…

Deriving sinking fund..

1) The future value of a terminating series of payments
2) Solve for a..
   1) Multiply both sides by i
   2) Divide both sides by $(1+i)^n -1$
   3) Rearrange…

$$V_n = a\left[\frac{(1+i)^n - 1}{i}\right]$$

$$V_n i = a(1+i)^n - 1$$

$$\frac{V_n i}{(1+i)^n - 1} = a$$

Sinking fund formula $$a = V_n\left[\frac{i}{(1+i)^n - 1}\right]$$

Using the sinking fund formula…

Example: You want to have $10,000 saved in 4 years. Your bank pays you an annual interest rate of 3% per year, compounded monthly. How much must you deposit each month?

$$a = V_n\left[\frac{i}{(1+i)^n - 1}\right]$$

V_n = $10,000
n = 48 (months)
i = 0.03/12 = 0.0025

$$a = \$10{,}000\left[\frac{0.0025}{(1+0.0025)^{48} - 1}\right] = \$196.34$$

Capital recovery formula

- Also known as the installment payment formula
- You know the present value (V_0), the number of payments, and the cost of capital (i), but you want to calculate the payment amount!
  - Commonly used for purchasing homes, vehicles, large real property

Deriving the capital recovery formula…

The present value of a terminating series of annual payments
(payments occur in each discounting period)

$$V_0 = a\left[\frac{(1+i)^n - 1}{(1+i)^n i}\right]$$

Solve for the payment amount by:

$$V_0(1+i)^n i = a[(1+i)^n - 1]$$

A) Multiplying both sides of the equation by $(1+i)^n i$
B) Dividing both sides of the equation by $[(1+i)^n - 1]$
C) Rearrange…

$$V_0\left[\frac{(1+i)^n i}{(1+i)^n - 1}\right] = a$$

$$a = V_0\left[\frac{(1+i)^n i}{(1+i)^n - 1}\right]$$

Another form of capital recovery formula…

Present value of a terminating series of payments (alternative form)

$$V_0 = a\left[\frac{1-(1+i)^{-n}}{i}\right]$$

Capital recovery equation

$$a = V_0\left[\frac{i}{1-(1+i)^{-n}}\right]$$

Using capital recovery equation

- If a logger buys a new tracked feller for $350,000 and has a cost of capital of 10%, what annual production from this loader is required to retire the debt in three years, assuming net revenue per ton is $15?

Step 1: Determine annual income required to retire the debt…

$$a = V_0\left[\frac{(1+i)^n i}{(1+i)^n - 1}\right] \qquad a = V_0\left[\frac{i}{1-(1+i)^{-n}}\right]$$

$$a = \$350{,}000\left[\frac{(1+0.10)^3 0.10}{(1+0.10)^3 - 1}\right] = \$140{,}740 \qquad a = \$350{,}000\left[\frac{0.10}{1-(1+0.10)^{-3}}\right] = \$140{,}740$$

Step 2: Determine the tons of production needed…

$140,740 / $15/ton = 9383 tons per year

Test your knowledge

Your business operates out of an old building that will need replacing in 15 years. If you can invest at 8%, compounded annually, how much must you reserve each year from your company's income to have the $500,000 necessary to replace the building?

$$a = V_n\left[\frac{i}{(1+i)^n - 1}\right] = \$500{,}000\left[\frac{0.08}{(1+0.08)^{15} - 1}\right] = \$18{,}415$$

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Test your knowledge

You are buying a new truck. The purchase price is $28,000. The dealer offers you 48 payments at 6.8% annual interest, compounded monthly, or you can make 60 payments at 7.25%, compounded monthly. What are your two payment choices? How much total finance charge will you pay in each case?

Use $a = V_0\left[\frac{i}{1-(1+i)^{-n}}\right]$ or $a = V_0\left[\frac{(1+i)^n i}{(1+i)^n - 1}\right]$

48 payments at 6.8%...

$$a = \$28{,}000\left[\frac{\frac{0.068}{12}}{1-\left(1+\frac{0.068}{12}\right)^{-48}}\right] = \$667.90 \qquad a = 28{,}000\left[\frac{\left(1+\frac{0.068}{12}\right)^{48}\frac{0.068}{12}}{\left(1+\frac{0.068}{12}\right)^{48} - 1}\right] = \$667.90$$

$667.90 x 48 = $32,059.20 total payments, finance charges = $4059.20

60 payments at 7.25%...

$$a = \$28{,}000\left[\frac{\frac{0.0725}{12}}{1-\left(1+\frac{0.0725}{12}\right)^{-60}}\right] = \$557.74 \qquad a = 28{,}000\left[\frac{\left(1+\frac{0.0725}{12}\right)^{60}\frac{0.0725}{12}}{\left(1+\frac{0.0725}{12}\right)^{60} - 1}\right] = \$557.74$$

$557.74 x 60 = $33,464.40 total payments, finance charges = $5464.40

Next lecture…

Choosing equations and preparing for quiz number one!