Lecture 8. Present value of a perpetual series of payments

FOR 2022. Financial Analysis for Natural Resources.

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Present value of a perpetual annual series

- Common in agriculture and forestry
  - Annual row crops
  - Hunting leases, taxes, administration costs
  - Annual income from forests may include nuts/seeds, berries, wildlife, WATER

Deriving the present value of a perpetual series of annual payments…

We already have this equation for the present value of a series of terminating annual payments…

$$V_0 = a\left[\frac{(1+i)^n - 1}{(1+i)^n i}\right]$$

Substituting ∞ for n…

$$V_0 = a\left[\frac{(1+i)^\infty - 1}{(1+i)^\infty i}\right]$$

And since $(1+i)^\infty - 1 \approx (1+i)^\infty$

We can write… $V_0 = \frac{a}{i}$ when n = infinity

Equation for present value of an perpetual annual series

$$V_0 = \frac{a}{i}$$

where, a = annual payment
i = interest rate

Using the present value of a perpetual series of annual payments

- If a forest of 500 acres yields annual timber harvests valued at $60,000 and your interest rate (cost of capital, hurdle rate) is 8%, what is the value of this 500 acres?

$$V_0 = \frac{a}{i}$$

$$= \frac{\$60{,}000}{0.08} = \$750{,}000$$

Using the present value of a perpetual series of annual payments

- If a farmer's field yields annual net returns of $160 per acre, what is the value of the land assuming an 8% cost of capital?
  - Hint: assume annual crops can be planted indefinitely on this land

$$V_0 = \frac{a}{i}$$

$$= \frac{\$160}{0.08} = \$2{,}000 / acre$$

Deriving present value of a perpetual series of periodic payments

- Common in forestry
- Rotations and cutting cycles are periodic rather than annual
- Assume that "r" years occur between net returns of "a" dollars

Present value of year r payment $= a\frac{1}{(1+i)^r}$

Present value of year 2r payment $= a\frac{1}{(1+i)^{2r}}$

Present value of year 3r payment $= a\frac{1}{(1+i)^{3r}}$

Present value of year infinity payment $= a\frac{1}{(1+i)^{\infty}}$

Deriving the present value of a perpetual series of periodic payments…

The sum of present value of all payments is… $V_0 = \frac{a}{(1+i)^r} + \frac{a}{(1+i)^{2r}} + \frac{a}{(1+i)^{3r}} + \dots + \frac{a}{(1+i)^{\infty}}$

Since each term is the preceding term multiplied by $\frac{1}{(1+i)^r}$ the equation is a geometric series which has a general form:

$$s = \frac{b(1-t^m)}{1-t}$$

If we define: $s = V_0$ $\quad b = \frac{a}{(1+i)^r}$ $\quad t = \frac{1}{(1+i)^r}$ $\quad m = \infty$

Substituting and simplifying since $\left(\frac{1}{(1+i)^r}\right)^{\infty} = 0$

We derive… $V_0 = \frac{\frac{a}{(1+i)^r}[1-0]}{1-\frac{1}{(1+i)^r}} = a\frac{1}{(1+i)^r - 1}$ ⇨ $V_0 = a\frac{1}{(1+i)^r - 1}$

Using the perpetual series of periodic payments

- A) At 10% interest, what is the present value of bare land for growing successive crops of pine that yields $5000 every 30 years?
- B) What if your time preference for resources was 6%?
- C) What if the rotation length at 6% was shortened to 28 years?

$$V_0 = a\frac{1}{(1+i)^r - 1}$$

A) $$V_0 = \$5000\frac{1}{(1+0.10)^{30} - 1} = \$303.96$$

B) $$V_0 = \$5000\frac{1}{(1+0.06)^{30} - 1} = \$1054.08$$

C) $$V_0 = \$5000\frac{1}{(1+0.06)^{28} - 1} = \$1216.05$$

Value of bare land

- The present value of a perpetual series of payments is often referred to as “bare land value”
  - Value of an infinite series of payments (annual or periodic) starting with bare land
  - Assumes that annual returns are sustainable
    - Balance between new technologies and any potential decline in productivity
    - May be conservative for this reason
- Rotation length and interest rates have a tremendous impact on bare land values!

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Test your knowledge: a decision situation

- Option A) You have 40-acres of land that you can rent for pasture at \$35 per acre per year, or
- Option B) You can grow trees on this land, which will yield net returns of \$6000 every 35 years.
- Which option has the higher land value at the following costs of capital?

	Pasture	Timber
4%	$V_0 = \frac{\$35}{0.04} = \$875$	$V_0 = \$6000\frac{1}{(1+0.04)^{35}-1} = \$2036.60$
8%	$V_0 = \frac{\$35}{0.08} = \$437.50$	$V_0 = \$6000\frac{1}{(1+0.08)^{35}-1} = \$435.24$
12%	$V_0 = \frac{\$35}{0.12} = \$291.67$	$V_0 = \$6000\frac{1}{(1+0.12)^{35}-1} = \$115.83$

Next lecture…

Sinking fund and capital recovery equations…