Lecture 7. Future and present value of a terminating series of periodic payments

FOR 2022. Financial Analysis for Natural Resources.

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Periodic terminating series

- Over the next 15 years, you will receive $500 every three years
  - At the end of years 3, 6, 9, 12, and 15
- While not common, these periodic terminating series are not difficult to calculate

Deriving the formula for future value of a terminating series of periodic payments

Assumptions:

a = the amount of periodic payment

t = number of interest bearing periods in a period or rotation

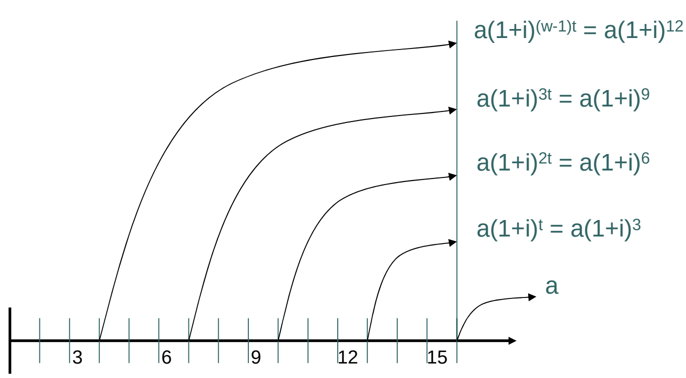
w = total number of periods or rotations

So the future value of:

payment in year t*w:	$= a$
payment in year t (w-1)	$= a(1+i)^t$
payment in year t (w-2)	$= a(1+i)^{2t}$
.	
.	
.	
first periodic payment	$= a(1+i)^{(w-1)t}$

Time line of future value of terminating series of periodic payments

5 payments (w) at three year intervals (t):



Deriving the future value of a terminating series of periodic payments

The future value V_n of all payments is:

$$V_n = a + a(1+i)^t + a(1+i)^{2t} + a(1+i)^{3t} + ... + a(1+i)^{(w-1)t} \quad [1]$$

Multiply both sides of the above equation by $(1+i)^t$:

$$V_n(1+i)^t = a(1+i)^t + a(1+i)^{2t} + a(1+i)^{3t} + a(1+i)^{4t} + ... + a(1+i)^{wt} \quad [2]$$

Subtract equation [1] from equation [2]

$$\begin{aligned} V_n(1+i)^t &= a(1+i)^t + a(1+i)^{2t} + a(1+i)^{3t} + a(1+i)^{4t} + ... + a(1+i)^{wt} \\ -(V_n &= a + a(1+i)^t + a(1+i)^{2t} + a(1+i)^{3t} + ... + a(1+i)^{(w-1)t}) \end{aligned}$$

$$V_n(1+i)^t - V_n = -a + a(1+i)^{wt} \quad [3]$$

Simplifying and factoring equation [3]…

$$V_n[(1+i)^t - 1] = a[(1+i)^{wt} - 1] \quad [4]$$

Divide both sides by $[(1+i)^t-1]$

$$V_n = a\frac{(1+i)^{wt} - 1}{(1+i)^t - 1} \quad [5]$$

Using the formula…

The future value of receiving $500 every three years for 15 years at an interest rate of 9% is:

- t = 3 years
- w = 5 (five rotations)
- a = $500
- i = 9% (0.09)

$$V_n = a\frac{(1+i)^{wt} - 1}{(1+i)^t - 1}$$

$$V_n = \$500\frac{(1+.09)^{15} - 1}{(1+0.09)^3 - 1} = \$4478.34$$

Another example…

- At 7% interest, a 40-acre Christmas-tree farm produces $10,000 of net revenue every 7 years. What is the future value of this cash flow after 5 rotations?

$$V_n = \$10{,}000 \frac{(1+.07)^{35} - 1}{(1+0.07)^{7} - 1} = \$159{,}737.16$$

Deriving present value of a terminating series of periodic payments

We know that present and future value are related by the future value equation, which in this case can be written as:

$$V_n = V_0 (1+i)^{wt}$$

Substituting this into the equation for future value of a terminating series of periodic payments we get:

$$V_0 (1+i)^{wt} = a \frac{(1+i)^{wt} - 1}{(1+i)^{t} - 1}$$

And solving for V_0 we get:

$$V_0 = a \frac{(1+i)^{wt} - 1}{[(1+i)^{t} - 1](1+i)^{wt}}$$

Using the equation for present value of a terminating series of periodic payments

- What is the present value at 9% interest of receiving $500 every three years for 15 years?

$$V_0 = \$500 \frac{(1+0.09)^{5x3} - 1}{[(1+0.09)^3 - 1](1+0.09)^{5x3}} = \$1229.48$$

Test your skills

- Our 40-acre Christmas tree farm earns net returns of $10,000 every seven years. After 5 rotations, what is the present value of the farm at 7% cost of capital?

$$V_0 = \$10{,}000 \frac{(1+0.07)^{35} - 1}{[(1+0.07)^7 - 1](1+0.07)^{35}} = \$14{,}961.45$$

Next lecture….

The present value of a perpetual series of payments!