Lecture 6. Future and present value of a terminating series of payments

FOR 2022. Financial Analysis for Natural Resources.

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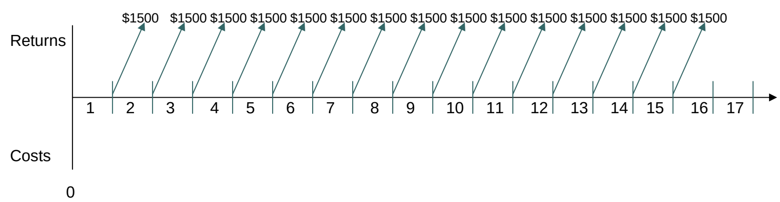
A terminating series of payments

- A series of payments that have some ending point in the future
  - Same amount each discount period (annual)
- Examples:
  - Taxes
  - Annual management fees
  - Rent
  - Salary
- You can use a “brute force” approach and use the single payment equations to determine the present or future value of a terminating series of payments

Future value of a terminating series of payments

- Every year on December 31, you deposit \$1500 into a savings account. At the end of 15 years you would have…



$$V_n = \$1500(1+i)^0 + \$1500(1+i)^1 + \$1500(1+i)^2 + ... \$ + 1500(1+i)^{13} + \$1500(1+i)^{14}$$

Deriving future value of a terminating series of payments

1. Generally, we can write the future value of n payments as…

$$V_n = a + a(1+i) + a(1+i)^2 + ... + a(1+i)^{n-1} \qquad [1]$$

2. Multiply both sides by (1+i)…

$$V_n(1+i) = a(1+i) + a(1+i)^2 + ... + a(1+i)^n \qquad [2]$$

3. Subtract equation [1] from equation [2] to obtain…

$$\begin{array}{rl} V_n(1+i) &= a(1+i) + a(1+i)^2 + ... + a(1+i)^n \\ -V_n &= -a - a(1+i) - a(1+i)^2 - ... - a(1+i)^{n-1} \\ \hline V_n(1+i) - V_n &= -a + a(1+i)^n \end{array} \qquad [3]$$

4. Factor the results of step 3…

$$V_n + V_n i - V_n = a(1+i)^n - a$$

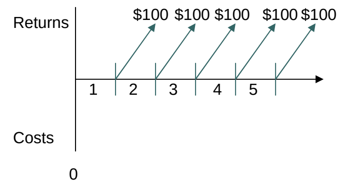
$$V_n(i) = a[(1+i)^n - 1] \qquad [4]$$

5. Solve for V_n by dividing both sides of equation [4] by i…

$$V_n = a\left[\frac{(1+i)^n - 1}{i}\right] \qquad [5]$$

Let's test our new formula

- What is the future value of 5 payments of $100 if our interest rate is 5%

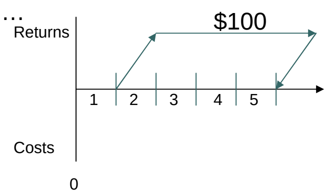


Brute force approach…

$$V_5 = \$100 + \$100(1.05)^1 + \$100(1.05)^2 + \$100(1.05)^3 + \$100(1.05)^4$$
$$V_5 = \$552.56$$

New future value of terminating series equation…

$$V_5 = \$100\left[\frac{1.05^5 - 1}{0.05}\right] = \$552.56$$



Deriving the present value of a terminating series of payments

We know that the future value of terminating series is:

$$V_n = a\left[\frac{(1+i)^n - 1}{i}\right] \qquad [1]$$

We know relationship between future and present value:

$$V_n = V_0(1+i)^n \qquad [2]$$

Substituting equation [2] into equation 1 and simplifying we get…

$$V_0(1+i)^n = a\left[\frac{(1+i)^n - 1}{i}\right]$$
$$V_0 = a\left[\frac{(1+i)^n - 1}{(1+i)^n i}\right] \qquad [3]$$

Present value of a terminating series of payments

$$V_0 = a\left[\frac{(1+i)^n - 1}{(1+i)^n i}\right]$$

We can multiply the RHS by $\frac{(1+i)^{-n}}{(1+i)^{-n}}$ to obtain a different form…

$$V_0 = a\left[\frac{(1+i)^n - 1}{(1+i)^n i}\right]\left[\frac{(1+i)^{-n}}{(1+i)^{-n}}\right]$$

$$V_0 = a\left[\frac{1-(1+i)^{-n}}{i}\right]$$

Solving for future value of a terminating series of payments

- Your landscaping business buys $2000 of peat each year to maintain its nursery. If you find a peat deposit on your property with an estimated supply of 10 years, what will your accumulated savings be, assuming a 12% interest rate?

$$V_n = \$2000\left[\frac{(1+0.12)^{10} - 1}{0.12}\right] = \$35097.47$$

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Solving for present value of a terminating series of payments

- The KP Hunt Club has signed a contract for \$1500/year for 10 years for a hunting lease. Assuming the club has a cost of capital of 9%, what would the equivalent, lump sum payment be at the beginning of the lease?

$$V_0 = 1500\left[\frac{(1+0.09)^{10}-1}{(1+0.09)^{10}0.09}\right] = \$9626.48$$

$$V_0 = \$1500\left[\frac{1-(1+0.09)^{-10}}{0.09}\right] = \$9626.49$$

Next lecture…

- Future and present value of a terminating series of periodic payments