



Lecture 4. Future and present value of a single payment.

FOR 2022. Financial Analysis for Natural Resources.



School of Forest Resources



The future value of a single payment

| Period | Value at end of period... |
|-----------|--|
| 1 | $V_1 = V_0 + V_0 i$ $V_1 = V_0 (1+i)$ |
| 2 | $V_2 = V_1 + V_1 i$ $= V_0 (1+i) + V_0 (1+i) i$ $= V_0 (1+i)(1+i)$ $= V_0 (1+i)^2$ |
| 3 | $V_3 = V_2 + V_2 i$ $= V_0 (1+i)^2 + V_0 (1+i)^2 i$ $= V_0 (1+i)^2(1+i)$ $= V_0 (1+i)^3$ |
| Generally | $V_n = V_0(1+i)^n$ where: V_0 = initial value V_n = future value n = number of discount periods i = interest rate |

Table adapted from Gunter and Haney, 1984.

● ● ● | Future value equation

$$V_n = V_0(1 + i)^n$$

- where:
 - V_n = future value after n discount periods
 - V_0 = initial or present value
 - n = number of discount or compounding periods
 - i = interest rate applicable to discount period

● ● ● | Calculating future value of a single payment

- If you invest \$1000 in a retirement fund that returns 9% per year over 50 years, what will be the future value of that amount?

$$V_{50} = \$1000(1.09)^{50} = \$74,357.52$$

- How much will the retirement be worth if you were to keep the investment for another 10 years (to 60 years)?

$$V_{60} = \$1000(1.09)^{60} = \$176,031.29$$



Deriving the present value equation

1. Since the future value equation is...

$$V_n = V_0(1+i)^n$$

2. We can manipulate it to find the present value by dividing both sides of the equation by $(1+i)^n$

$$\frac{V_n}{(1+i)^n} = V_0$$

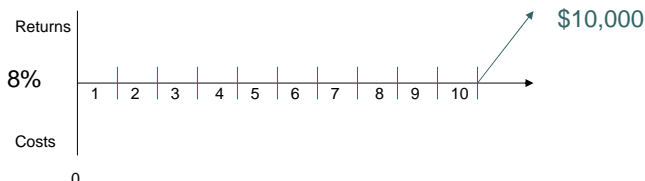
2. This can be written in two ways...

$$V_0 = \frac{V_n}{(1+i)^n} \quad \text{Or, since} \quad \frac{1}{x^n} = x^{-n} \quad \Rightarrow \quad V_0 = V_n(1+i)^{-n}$$



Using the present value equation...

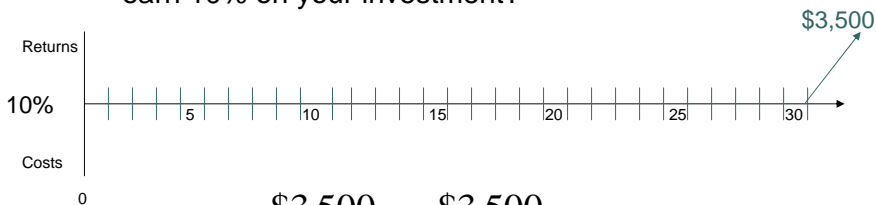
- o Your distant, great uncle has passed away and has left you \$10,000. The only problem is, his will stipulates that you cannot receive this money until you are 30 years old. If you are currently 20, and you have a time preference for resources of 8%, what is this worth to you in today's dollars?



$$V_0 = \frac{\$10,000}{(1.08)^{10}} = \frac{\$10,000}{2.1589} = \$4631.99$$

Using the present value equation...

- A pine plantation will return a net return of \$3500 per acre in 30 years. How much can you afford to pay for establishing the plantation if you want to earn 10% on your investment?



$$V_0 = \frac{\$3,500}{(1.10)^{30}} = \frac{\$3,500}{17.4494} = \$200.58$$

The “multiplier”

- You will sometimes hear about the “multiplier”
- In general, it is the value that is multiplied with a present (future) value to get a future (present) value.
- Future value multipliers are larger than one. For single payment, the multiplier is simply: $(1+i)^n$
 - So, for an interest rate of 6% and $n = 25$, the multiplier is $(1.06)^{25} = 4.2919$
 - So, for every \$1 invested in year 0 at 6%, you will have \$4.2919 dollars at the end of year 25.
- Present value multipliers are less than one and are calculated by $(1+i)^{-n}$:
 - For an interest rate of 6% and each dollar received in 25 years, the “multiplier” is $(1.06)^{-25} = 0.2330$
 - If your time preference for money is 6%, then every dollar received 25 years from now is worth \$0.23 today.




The rule of 72....

- The length of time it will take for investment to roughly double in value can be approximated by dividing the compound interest rate, expressed as a percent, into 72.
 - Thus, at 6%, an investment will approximately double in $72/6 = 12$ years.
 - Note that $1.06^{12} = 2.0122$
 - At 18%, an investment will approximately double in about 4 years



Practice using the present and future value equations

- If you buy a \$100 U.S. savings bond for \$50 (typically bought at $\frac{1}{2}$ face value) for your nephew when he is born, what will the bond be worth in 18 years when he starts college. Assume that the bond earns 4% interest per year, guaranteed.



Practice using the present and future value equations

- You have a 23 year-old shortleaf pine plantation in northwest Arkansas. Your growth and yield estimates show that you will receive \$2800 per acre for the harvest in 22 years, according to your forest plan. Your brother offers to buy the rights to the timber today, and wants to know how much you would ask per acre for the timber today. You believe you could invest whatever your brother pays you at 9% interest for 22 years.



Next lecture...

- Working with spreadsheets...