Lecture 2. All about interest

FOR 2022. Financial Analysis for Natural Resources.

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What is the interest rate?

- A quantitative or mathematical expression of an entities time preference for money or resources
  - A bank "pays" you 2% interest per year on your savings account
  - You pay a bank 14% interest per year on the balance on your credit card

Interest vs. principal

- If you make a deposit into the bank of $100, this is the *principal*
  - Likewise, if you borrow $150,000 to buy a home, this is also called the principal
- Interest is the amount of money paid to the lender (saver) for the time they have allowed anther entity to use that money.

Simple interest

- Interest charges are calculated only on the principal amount
- For example, a savings account of $1000 with a simple annual interest of 5%

	Year 1	Year 2	Year 3
Principal	$1000	$1000	$1000
Annual Interest	$50	$50	$50
Total principle + interest	$1050	$1100	$1150

Compound interest

- Interest is paid on principal and any accumulated interest from previous time periods
- For example, $1000 deposited in a bank at 5% interest, compounded annually:

	Year 1	Year 2	Year 3
Principal + accumulated interest	$1000	$1050	$1102.5
Annual Interest	$1000 x 0.05 = $50	$1050 x 0.05 = 52.5	$1102.5 x 0.05 = 55.13
Total principle + interest	$1050	$1102.5	$1157.63

Let's look at this from the borrower's perspective

- You borrow $150,000 from a mortgage company to buy a house.
- The mortgage company asks for 6% interest, compounded annually
- How would interest accumulate if you were to pay this debt off at the end of 15 years? Let's see on the next slide…

15-year interest accumulation

Year	Principal + Accumulated Interest	Interest Charges
1	150,000	9,000
2	159,000	9,540
3	168,540	10,112.40
4	178,652.40	10,719.14
5	189,371.54	11,362.29
6	200,733.83	12,044.03
7	212,777.86	12,766.67
8	225,544.53	13,532.67
9	239,077.20	14,344.63
10	253,421.83	15,205.31
11	268,627.14	16,117.63
12	284,744.77	17,084.69
13	301,829.46	18,109.77
14	319,939.23	19,196.35
15	339,135.58	20,348.14
Total	359,482.72	

Difference between simple and compound interest

- Mortgage example
  - Simple interest after 15 years is $135,000
  - Compound interest after 15 years is $209,483
- Compound interest most appropriate to use because lender should receive time value for unpaid interest in each time period

A mathematical expression of interest

- An interest rate can be expressed as a percentage
  - 4%
- Or as a decimal
  - 0.04 = 4%
- Interest rates are a function of three factors:
  - Pure time value for resources (*i*)
  - Risk (*r*)
  - Inflation (*f*)

Determining an interest rate

- For a particular entity (person or organization) is considering a long-term, multi-year project:
  - Let's say the pure, risk-free time preference for resources is 4%
  - They anticipate that the risk of projects failing each year is 1%
  - They expect inflation to raise the cost of future goods and services over the project lifespan to be 3%

Determining an interest rate

- The interest rate that should be applied is:
  $$rate = [(1+i)(1+r)(1+f)-1]100$$
  - i = pure time preference for money
  - r = risk rate
  - f = inflation rate
- So, in our instance:
  $$rate = [(1+0.04)(1+0.01)(1+0.03)-1]100 = 8.19\%$$
- You cannot simply “add” the rates because they compound with each other!

Compound interest components…

- To correctly apply compound interest rate, you must know:
  - Rate
  - Time period to which rate applies
- For example, you can have:
  - 12% compounded annually
  - 1% compounded monthly
  - The two rates are NOT equivalent!
  - If 1% is compounded monthly for 12 months, the effective annual rate is 12.68%
  - This can make tremendous difference over long time periods
- Discrete compounding vs. continuous!
  - Discrete – finite number of discount periods
  - Continuous – infinite number of compounding periods

Adjusting for time periods

- You can have a compound interest rate stated as the following:
  - 6% annual interest compounded monthly
    - That means that each month, the principle + unpaid interest or total balance will be charged 6%/12 = ½ % or 0.005 to calculate the new monthly interest charges
    - Effectively, 6% annual interest compounded monthly is the same as:

$$\left[\left(1+\frac{0.06}{12}\right)^{12}-1\right]100 = 6.1678\%$$

Formula for determining effective annual interest rate

$$\left[\left(1+\frac{i}{n}\right)^{n}-1\right]100 = effective\_annual\_rate$$

- i = nominally state annual rate
- n = number of compounding periods within the year

- So, what is the effective annual rate for:
  - 12% annual interest, compounded:
    - **Quarterly**
      - 12.5509%
    - **Monthly**
      - 12.6825%
    - **Daily** (assume 365.25 days per year)
      - 12.7475%

Continuous compounding

- Effective rate is solved by:
- So, if our nominal annual rate is 12%, compounded continuously, the effective rate is:

$$e^{i} - 1$$

$$e^{.12} - 1 = 0.127497$$

or 12.75%

Next lecture…

We will discuss using a time line and financial analysis conventions….