


Lecture 18. Methods for estimating sustainable cutting levels

FOR 4684 Natural Resource Economics and Management



$$AC = \frac{\text{Acres}}{\text{Rotation_Age}}$$



Determining the allowable cut (AC)

- Major considerations:
 - Determine the volume that is to be cut
 - Determine some cutting sequence of stand (simple harvest schedule)
 - Species, size and quality
 - Spatial arrangement of cut
- Answers to these questions depends upon:
 - Management objectives
 - Timber markets
 - Silvicultural needs
 - Logging problems
 - Degree of harvest continuity desired



Determining allowable cut

- Timber is a crop that seldom requires immediate harvest
- Approaches to determining AC:
 - Area control
 - Volume control
 - Combined area/volume control
 - Modern operations research techniques such as linear programming



Area control at its most simple

- Principle is to determine volume to be cut based on acres assigned for cutting
- Specifically,

$$AC = \frac{\textit{Acres}}{\textit{Rotation_Age}}$$



Characteristics of simple area control

- Advantages
 - Very simple
 - Easily understood
 - Results in normal forest age class distribution after one rotation
- Problems
 - Even flow of timber volume not guaranteed

$$AC = \frac{A}{R}$$




Example of area control

- We'll look at red pine in Minnesota
- 1600 acre forest
- Three major compartments

Stand	Age	Acres	Stocking	Site Index
A	80	700	90%	65
B	40	300	80%	45
C	60	600	85%	75

Need yield table information...



Stand Age	Site Index				
	45	55	65	75	85
30	888	1283	1743	2266	2850
40	1483	2142	2910	3784	4760
50	2017	2914	3959	5147	6475
60	2476	3577	4860	6319	7949
70	2866	4142	5627	7316	9203
80	3199	4623	6280	8165	10272
90	3485	5035	6841	8894	11189
100	3731	5392	7325	9523	11981
110	3946	5702	7746	10071	12670
120	4134	5974	8116	10552	13274
130	4301	6214	8442	10976	13809
140	4449	6428	8732	11353	14283
150	4581	6619	8992	11691	14708

$$Y = 6.4088S^{1.8342} e^{\left(\frac{-61.534}{A}\right)}$$



Area Control Step 1

- Determine acres per age class under regulation assuming equal site productivity

$$= 1600 \text{ acres} / 80 \text{ years} = 20 \text{ acres per age class}$$



Area Control Step 2

- Find fully-stocked yield per acre at rotation age for each stand

Stand	Yield per acre at 80 years
A	6,280 ft. ³
B	3,199 ft. ³
C	8,165 ft. ³



Area Control Step 3

- Find average productivity weighted by acres

$$= \frac{6,280(700) + 3,199(300) + 8,165(600)}{1600} = 6,409 \text{ cubic feet}$$



Area Control Step 4

- Find areas of equal productivity (acres to cut per year)

$$A = \left(\frac{6409}{6280} \right) 20 = 20.4$$

$$B = \left(\frac{6409}{3119} \right) 20 = 40.1$$

$$C = \left(\frac{6409}{8165} \right) 20 = 15.7$$



Area Control Step 5

- Find cutting time per stand
 - A: 700 acres / 20.4 acres/year = 34.3 years
 - B: 300 acres / 40.1 acres/year = 7.5 years
 - C: 600 acres / 15.7 acres/year = 38.2 years
- Note that total cutting time for all stands equals rotation age of 80 years!



Area Control Step 6

- Assign cutting sequence for stands and find average age at harvest
 - Assume we will cut oldest stands first, so cutting sequence is stand A, C, B.

$$A = 80 + \frac{34.3}{2} = 97$$

$$C = 60 + 34.3 + \frac{38.2}{2} = 113$$

$$B = 40 + 34.3 + 38.2 + \frac{7.5}{2} = 116$$



Area Control Step 7

- Determine the volume harvested (per year) in first rotation

$$A = 7187 \times 20.4 \times 0.90 = 131.95 \text{ Mcf}$$

$$C = 10,222 \times 15.7 \times 0.85 = 136.41 \text{ Mcf}$$

$$B = 4062 \times 40.1 \times 0.80 = 130.31 \text{ Mcf}$$



Area Control Step 8

- Determine volume harvested per year for future rotations

$$A = 6280 \times 20.4 \times 0.90 = 115.30 \text{ Mcf}$$

$$C = 8,165 \times 15.7 \times 0.90 = 115.37 \text{ Mcf}$$

$$B = 3,199 \times 40.1 \times 0.90 = 115.45 \text{ Mcf}$$



Allowable cut area control

- Advantages
 - Volume cut is average volume per unit area times acres
 - After one rotation forest is regulated
 - An approximate uniform cut can be obtained
 - Simple and direct
 - Areas to be cut can be identified on ground
 - Well suited for even-aged forests
- Disadvantages
 - Area control applied to an irregular forest will have irregular cuts in volume, size and quality
 - For largely mature forests, area control may cause great losses from holding mature stands too long
 - Volumes harvested are not explicitly considered.



A second example...

- Using yield table for shortleaf pine
- 900 acre forest
- Rotation length of 45 years
- Three stands,
 - A: 150 acres, 15 years old, SI = 90, 100% stocked
 - B: 450 acres, 30 years old, SI = 70, 80% stocked
 - C: 300 acres, 45 years old, SI = 60, 90% stocked
- All stands will be 100% stocked in future



Yield table for shortleaf pine

Age (years)	Site index in feet at 50 years						
	40	50	60	70	80	90	100
	Yield in board feet (International c -inch rule) per acre						
15	--	--	--	--	--	200	1400
20	--	--	--	250	1600	3800	7500
25	--	--	800	2800	5700	10500	14950
30	--	900	3300	6900	11550	17000	23050
35	600	2800	6600	11750	17100	23750	30900
40	1750	5000	10200	16400	22450	29750	37600
45	3000	7500	13600	20250	27200	35300	43850
50	4200	9800	16600	23800	31600	40000	49000
55	5650	11900	19250	27000	35200	44200	53600
60	6900	13800	21500	29700	38200	47500	57300
65	8100	15400	23600	3200	40800	50400	60800
70	9200	16900	25300	34100	43200	53200	63750
75	10150	18200	26900	35900	45300	55500	66250
80	11050	19400	28300	37700	47250	57700	68400
85	11900	20500	29600	39200	49000	59500	70300
90	12600	21400	30750	40700	50700	61000	72000
95	13300	22300	31850	42000	52200	62900	73500
100	14000	23100	32900	43300	53500	63700	74750



Shortleaf pine area control

- Step 1: Acres per age class under regulation assuming equal site productivity
 - $900/45 = 20$ acres
- Step 2: Yield per acre for each stand at rotation age:
 - A = 35,300 bf / ac
 - B = 20,250 bf / ac
 - C = 13,600 bf / ac



Shortleaf pine area control

- Step 3. Average productivity weighted by acres

$$= \frac{150(35300) + 450(20250) + 300(13600)}{900} = 20,542$$

- Step 4. Acres of equal productivity
 - A: $(20542/35300) \times 20 = 11.6$ acres
 - B: $(20542/20250) \times 20 = 20.3$ acres
 - C: $(20542/13600) \times 20 = 30.2$ acres



Shortleaf pine area control

- Step 5. Cutting time per stand
 - A: $150 \text{ acres} / 11.6 \text{ acres/year} = 12.9 \text{ years}$
 - B: $450 \text{ acres} / 20.3 \text{ acres/year} = 22.2 \text{ years}$
 - C: $300 \text{ acres} / 30.2 \text{ acres/year} = 9.9 \text{ years}$
- Step 6. Order of harvest and average age at harvest.
 - Order: C then B then A (oldest to youngest)
 - Average age at harvest:
 - C: $45 + (9.9/2) = 50 \text{ years}$
 - B: $30 + 9.9 + (22.2/2) = 51 \text{ years}$
 - A: $15 + 9.9 + 22.2 + (12.9/2) = 54 \text{ years}$



Shortleaf pine area control

- Step 7. Volume harvested per year in first rotation
 - C: $16,600 \times 30.2 \times 0.90 = 451,188 \text{ bf}$
 - B: $24,440 \times 20.3 \times 0.80 = 396,906 \text{ bf}$
 - A: $43,360 \times 11.6 \times 1.00 = 502,976 \text{ bf}$
- Step 8. Volume harvested per year in future rotations.
 - C: $13,600 \times 30.2 \times 1.00 = 410,720 \text{ bf}$
 - B: $20,250 \times 20.3 \times 1.00 = 411,075 \text{ bf}$
 - A: $35,300 \times 11.6 \times 1.00 = 409,480 \text{ bf}$

● ● ● | Next...

Volume control techniques for determining allowable cut

● ● ● | About volume control

- Volume control provides approximate estimate of volume that is applied for a short time (< 10% of rotation) and then recalculated
- Can be applied to even or uneven-aged stands (not dependent on yield tables)
- Allowable cut determined in volume, then acres must be found to provide that volume
- Will not guarantee a “regulated” state of the forest after one rotation
- Several formulas for volume control...



Von Mantel's

- Simple
- Doesn't consider whether or not current growing stock is desirable
- Should be applied only in well-regulated forests
- Assumes linear increase in growing stock with age

$$Y_a = AC = \frac{2G_a}{R - a}$$



Hundeshagen's

- Simple
- Doesn't consider whether or not current growing stock is desirable
- Should be applied only in well-regulated forests
- Need yield table
- Assumes ratio of growth to yield in actual forest is same as that in normal forest

$$\frac{Y_a}{G_a} = \frac{Y_R}{G_R}$$

$$Y_a = AC = \frac{Y_R}{G_R} G_a$$



Austrian formula

- Modifies allowable cut with some growing stock information
- Reduce or increases AC to move growing stock towards desired state
- Better in irregular conditions
- Best when growing stock is even-aged second-growth stands
- Assumes balanced stocking

$$AC = I + \frac{G_a - G_R}{a}$$

where:

I = current annual increment

a = number of years to adjust growing stock.



Chapman's formula

- Modifies allowable cut with some growing stock information
- Reduce or increases AC to move growing stock towards desired state over one rotation
- Better in irregular conditions
- Best when growing stock is even-aged second-growth stands
- Assumes balanced stocking

$$AC = I_e + \frac{G_a - G_R}{R}$$

where:

I_e = mean annual increment at rotation age and average stocking (MAI_R)



Example of volume control

- We'll look at the red pine example in Minnesota
- 1600 acre forest
- Three major compartments

Stand	Age	Acres	Stocking	Site Index
A	80	700	90%	65
B	40	300	80%	45
C	60	600	85%	75

Need yield table information...



Stand Age	Site Index				
	45	55	65	75	85
30	888	1283	1743	2266	2850
40	1483	2142	2910	3784	4760
50	2017	2914	3959	5147	6475
60	2476	3577	4860	6319	7949
70	2866	4142	5627	7316	9203
80	3199	4623	6280	8165	10272
90	3485	5035	6841	8894	11189
100	3731	5392	7325	9523	11981
110	3946	5702	7746	10071	12670
120	4134	5974	8116	10552	13274
130	4301	6214	8442	10976	13809
140	4449	6428	8732	11353	14283
150	4581	6619	8992	11691	14708

$$Y = 6.4088S^{1.8342} e^{\left(\frac{-61.534}{A}\right)}$$



Volume control calculations

- 1. Average weighted:
 - Site index (SI_w)
 - Stocking (S_w)
 - Age (A_w)
- 2. Actual growing stock volume (G_a)
- 3. Regulated growing stock volume (G_R)
- 4. Yield for regulated growing stock (Y_R)
- 5. Current annual increment or estimate using PAI
- 6. Mean annual increment at rotation age for the forest (I_e)



Average weighted site, stocking and age...

$$SI_w = \frac{65 \times 700 + 45 \times 300 + 75 \times 600}{1600} = 65$$

$$S_w = \frac{90\% \times 700 + 80\% \times 300 + 85\% \times 600}{1600} = 86\%$$

$$A_w = \frac{80 \text{ yrs} \times 700 + 40 \text{ yrs} \times 300 + 60 \text{ yrs} \times 600}{1600} = 65 \text{ yrs}$$

● ● ● | Actual growing stock volume...

$$\begin{aligned}
 G_a &= (6280 \times 90\% \times 700) \\
 &+ (1483 \times 80\% \times 300) \\
 &+ (6319 \times 85\% \times 600) \\
 &= 7,535,010 \text{ cubic feet}
 \end{aligned}$$

● ● ● | Regulated growing stock volume...

$$\begin{aligned}
 G_R = G_{80} &= 10 \left[1,743 + 2910 + 3959 + 4860 + 5627 + \frac{6280}{2} \right] \\
 &= 222,390 \text{ cubic feet on 80 acres} \\
 &= 222,390 \frac{1600}{80} \times 90\% = 4,003,020 \text{ cubic feet} \\
 &\hspace{15em} \text{on 1600 acres at future stocking of 90\%}
 \end{aligned}$$

● ● ● | Yield for regulated growing stock...

$$Y_R = 6280 \times \frac{1600}{80} \times 90\% = 113,040 \text{ cubic feet}$$

● ● ● | Current annual increment...

1. Take derivative of yield equation to find CAI:

$$CAI = \frac{\partial Y}{\partial A} = \frac{394.359SI^{1.8342}e^{\frac{-61.534}{A}}}{A^2}$$

2. Calculate CAI for each stand...

Stand	Age	CAI	CAI x acres x stocking
A	80	60.382	60.382 x 700 x 90%
B	40	57.016	57.016 x 300 x 80%
C	60	108.001	108.001 x 600 x 85%
Total			106,805 ft. ³



Alternative method for current annual increment...

Calculate CAI for average weighted
age-site-stocking combination

$$CAI = \frac{\partial Y}{\partial A} = \frac{394.359(65)^{1.8342} e^{\frac{-61.534}{65}}}{65^2} \times 86\% = 105,696$$

cubic feet



Estimating CAI from PAI...

$$PAI_A = \frac{(6,841 - 6,280)}{10} \times 700 \times 90\% = 35,343 \text{ ft.}^3$$

$$PAI_B = \frac{(2,017 - 1,483)}{10} \times 300 \times 80\% = 12,816 \text{ ft.}^3$$

$$PAI_C = \frac{(7,316 - 6,319)}{10} \times 600 \times 85\% = 50,847 \text{ ft.}^3$$

$$CAI \approx PAI_{Forest} = 35,343 + 12,816 + 50,847 = 99,006 \text{ ft.}^3$$



Mean annual increment at rotation age...

$$I_e = MAI_R = \frac{6,280}{80} \times 1600 \times 86\% = 108,330 \text{ ft.}^3$$



Summary of data...

- $G_a = 7,535,010 \text{ ft.}^3$
- $G_R = 4,003,020 \text{ ft.}^3$
- $Y_R = 113,040 \text{ ft.}^3$
- $CAI = 106,805 \text{ ft.}^3$
- $MAI_R = 108,330 \text{ ft.}^3$
- $A_w = 65 \text{ years}$
- $SI_w = 65 \text{ ft.}$
- $S_w = 86\%$



Von Mantel AC

$$AC = \frac{2G_a}{R} = \frac{2 \times 7,535,010}{80} = 188,375$$

$$AC = \frac{2G_a}{R-a} = \frac{2 \times 7,535,010}{80-30} = 301,400$$



Hundeshagen AC

$$AC = \frac{Y_R}{G_R} G_a = \frac{113,040}{4,003,020} 7,535,010 = 212,779$$

● ● ● | Austrian AC

$$AC = CAI + \frac{G_a - G_R}{a}$$

$$AC = 106,805 + \frac{7,535,010 - 4,003,020}{30} = 224,538$$

● ● ● | Chapman's AC

$$AC = MAI_R + \frac{G_a - G_R}{R}$$

$$AC = 108,330 + \frac{7,535,010 - 4,003,020}{80} = 152,480$$



Modified Barnes Method of Allowable Cut

- AC = yield at average cutting age times volume at average weighted stocking

1. Average cutting age

$$A_{wc} = \frac{R}{2} + A_w = 65 + \frac{80}{2} = 105$$

2. Yield at average stocking

$$Y_{105, SI=65} = 7542.4$$

3. Allowable cut =

$$AC = 7542.4 \times 86\% \times \frac{1600}{80} = 130,106$$



Next...

An example of allowable cut using tabular check and simulation



Tabular Check: Simulating Allowable Cut Solutions

- Start with some estimate of allowable cut
- Simulate allowable cut estimate on existing forest to determine actual cutting time for forest
- If sum of all cutting times is within some reasonable range from rotation age, stop.
 - If sum of cutting times is greater than rotation age, increase allowable cut and resimulate
 - If sum of cutting times is less than rotation age, reduce allowable cut and resimulate



Tabular Check on our red pine forest example

- Use AC estimate from Modified Barnes

Stand	Age	Acres	Stocking	Site Index
A	80	700	90%	65
B	40	300	80%	45
C	60	600	85%	75

- AC = 130 Mcf / year

$$Y = 6.4088S^{1.8342} e^{\left(\frac{-61.534}{A}\right)}$$

First Iteration: AC = 130 Mcf / year

Stand	Site Index	Stocking Percent	Present Age	Present Volume (Total)	Average Age at Harvest	Average Volume at Harvest	Cutting Time	
							Per stand	Cumulative
A	65	0.90	80	6.280 x 0.90 x 700 = 3956	$80 + \left(\frac{3956}{130}\right) / 2 = 95.2$ $80 + \left(\frac{34.4}{2}\right) = 97.2$ $80 + \left(\frac{34.9}{2}\right) = 97.4$	7.101 x 0.90 x 700 = 4474 7.196 x 0.90 x 700 = 4533 7.207 x 0.90 x 700 = 4540	4474 / 130 = 34.4 4533 / 130 = 34.9 4540 / 130 = 35.0	35.0
	700							
	75	0.85						
600								
B	45	0.80	40 + 75.4 = 115.4	4.051 x 0.80 x 300 = 972	$115.4 + \left(\frac{972}{130}\right) / 2 = 119.1$ $115.4 + \left(\frac{7.6}{2}\right) = 119.2$	4.119 x 0.80 x 300 = 988 4.120 x 0.80 x 300 = 989	988 / 130 = 7.6 989 / 130 = 7.6	75.4 + 7.6 = 83.0
	300		115.4	972	$115.4 + \left(\frac{7.6}{2}\right) = 119.2$	4.120 x 0.80 x 300 = 989	989 / 130 = 7.6	75.4 + 7.6 = 83.0
	700		115.4	972	$115.4 + \left(\frac{7.6}{2}\right) = 119.2$	4.120 x 0.80 x 300 = 989	989 / 130 = 7.6	75.4 + 7.6 = 83.0

Second Iteration: AC = 138 Mcf / year

Stand	Site Index	Stocking Percent	Present Age	Present Volume (Total)	Average Age at Harvest	Average Volume at Harvest	Cutting Time	
							Per stand	Cumulative
A	65	0.90	80	6.280 x 0.90 x 700 = 3956	$80 + \left(\frac{3956}{138}\right) / 2 = 94.3$ $80 + \left(\frac{32.2}{2}\right) = 96.1$ $80 + \left(\frac{32.6}{2}\right) = 96.3$	7.058 x 0.90 x 700 = 4447 7.145 x 0.90 x 700 = 4501 7.154 x 0.90 x 700 = 4506	4447 / 138 = 32.2 4501 / 138 = 32.6 4506 / 138 = 32.7	32.7
	700							
	75	0.85						
600								
B	45	0.80	40 + 70.2 = 110.2	3.949 x 0.80 x 300 = 947.9	$110.2 + \left(\frac{947.9}{138}\right) / 2 = 113.6$ $110.2 + \left(\frac{7.0}{2}\right) = 113.7$	4.017 x 0.80 x 300 = 964 4.018 x 0.80 x 300 = 964	964 / 138 = 7.0 964 / 138 = 7.0	70.2 + 7.0 = 77.2
	300		110.2	947.9	$110.2 + \left(\frac{7.0}{2}\right) = 113.7$	4.018 x 0.80 x 300 = 964	964 / 138 = 7.0	70.2 + 7.0 = 77.2
	700		110.2	947.9	$110.2 + \left(\frac{7.0}{2}\right) = 113.7$	4.018 x 0.80 x 300 = 964	964 / 138 = 7.0	70.2 + 7.0 = 77.2

Third Iteration: AC = 134 Mcf / year

Stand	Site Index	Stocking Percent	Present Age	Present Volume (Total)	Average Age at Harvest	Average Volume at Harvest	Cutting Time	
	Acres						Per stand	Cumulative
A	65	0.90	80	6.280 x 0.90 x 700 = 3956	$80 + \left(\frac{3956}{134}\right) / 2 = 94.8$ $80 + \left(\frac{33.3}{2}\right) = 96.6$ $80 + \left(\frac{33.7}{2}\right) = 96.9$	7.079 x 0.90 x 700 = 4460 7.169 x 0.90 x 700 = 4517 7.180 x 0.90 x 700 = 4523	$4460 / 134 = 33.3$ $4517 / 134 = 33.7$ $4523 / 134 = 33.8$	33.8
	700							
C	75	0.85	60 + 33.8 = 93.8	9.141 x 0.85 x 600 = 4662	$93.8 + \left(\frac{4662}{134}\right) / 2 = 111.2$ $93.8 + \left(\frac{38.6}{2}\right) = 113.1$ $93.8 + \left(\frac{38.9}{2}\right) = 113.3$	10.132 x 0.85 x 600 = 5167 10.226 x 0.85 x 600 = 5215 10.234 x 0.85 x 600 = 5220	$5167 / 134 = 38.6$ $5215 / 134 = 38.9$ $5220 / 134 = 39.0$	33.8 + 39.0 = 72.8
	600							
B	45	0.80	40 + 72.8 = 112.8	4.000 x 0.80 x 300 = 960	$112.8 + \left(\frac{960}{134}\right) / 2 = 116.4$ $112.8 + \left(\frac{7.3}{2}\right) = 116.4$	4.069 x 0.80 x 300 = 976 4.070 x 0.80 x 300 = 977	$976 / 134 = 7.3$ $977 / 134 = 7.3$	72.8 + 7.3 = 80.1
	300							

Stand	Site Index	Stocking Percent	Present Age	Present Volume (Total)	Average Age at Harvest	Average Volume at Harvest	Cutting Time	
	Acres						Per stand	Cumulative

● ● ● | Next lecture...

Forest planning concepts...