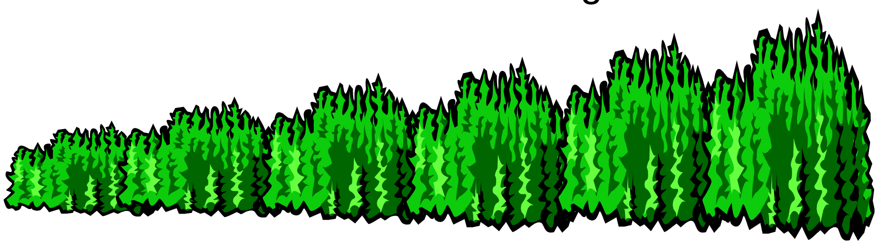
Lecture 17. Normal forest model and concepts of forest regulation

FOR 4684 Natural Resource Economics and Management



Forest regulation

- Organization and control of the growing stock of a forest for sustained yield
- Regulation tool is timing and size of timber harvests
- Answering the questions:
  - How much and when do I cut to create a sustainable forest?

Why try to regulate a forest?

- Yearly cuts of nearly equal acreage, volume, quality and value provide a stable planning base
- Balance between annual and periodic income and expenses
- Degree of safety from damage events from regular harvesting

Critical questions in forest regulation

- Sustained yield depends on:
  - Rotation length
  - Current and expected utilization standards
  - Current and expected technologies
  - Current yield and expected growth
- How should conversion from unregulated forest to regulated forest be accomplished
  - Should we allow larger cuts during conversion period or should we always enforce even-flow?

What is a “normal” forest?

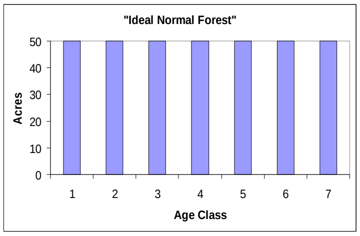
- A model of a forest age class structure
- Age and size classes are represented in such proportion and growing consistently at such rates that an approximate equal annual or periodic yield of products of desired size and quality are obtained.

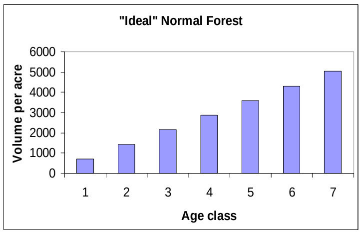
Normal forest model

- A very simple model
- Assumes “normality” in terms of:
  - Growth
  - Rotation
  - Age-class distribution
  - Growing stock
  - Cut
- Additional assumptions:
  - Linear growth
  - Equal site quality
  - Equal stocking of all stands with one species
  - No intermediate harvests

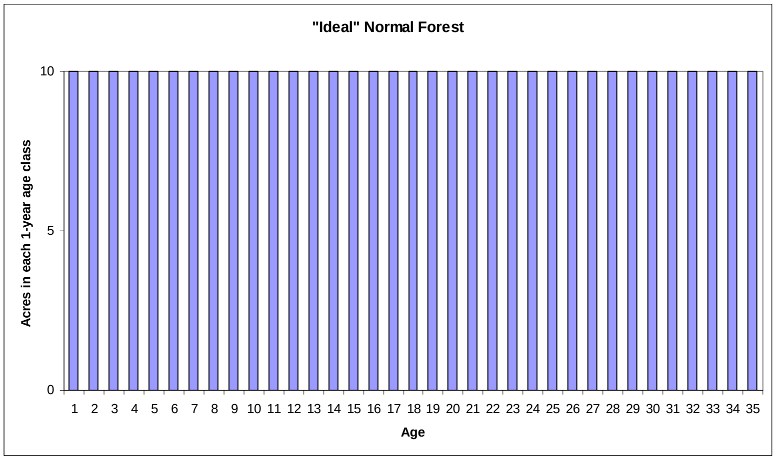
Graph of a normal forest

- Assume 35 year rotation
- Assume 350 acres
- 7 5-year age classes
- Volume and age class distributions in graphs to right are "normal"

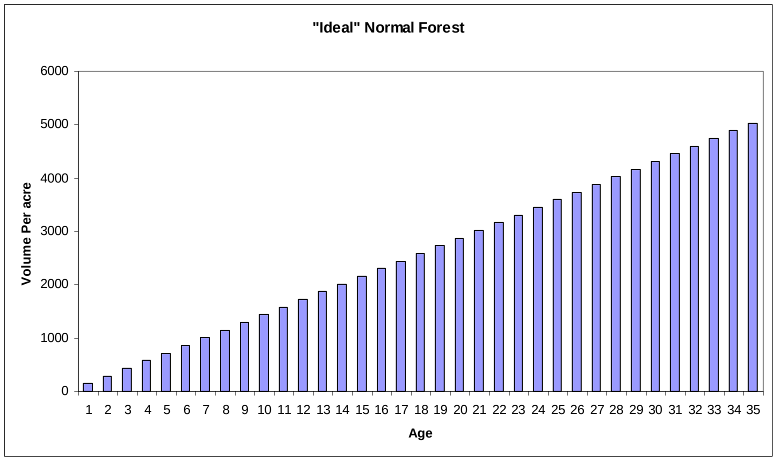




If we use 1-year age classes….



If we use 1-year age classes….



Let's define some terms

- G_i = growing stock volume in age class i
- R = rotation or age of oldest age class
- Y_i=growth of age class i

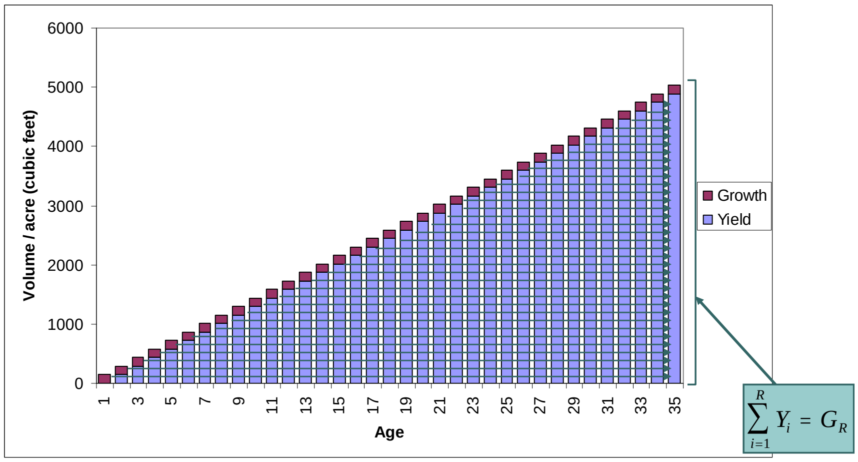
$$\sum_{i=1}^{R} G_i = G_s = total\_growing\_stock\_of\_forest$$

$$= \frac{R}{2} G_R \quad \text{(on R acres)}$$

$$\sum_{i=1}^{R} Y_i = total\_annual\_forest\_growth$$

$$= G_R$$

Sum of all growth = G_R



Annual cut of "ideal" normal forest

- Annual cut
  - Can harvest sum of all growth in forest

$$G_R = \sum_{i=1}^{R} Y_i = \frac{2\sum_{i=1}^{R} G_i}{R}$$

- Cut over one rotation
  - R x annual cut

$$= R \times G_R = 2\sum_{i=1}^{R} G_i$$

this is twice the growing stock volume!

Thus, in a regulated forest…

We will harvest, in one rotation, approximately twice the amount of volume that is standing on the forest at any given time!

Application to our "ideal" forest

- What is total growing stock of this forest?

Age Class	Mid point of age class	Acres	Volume
1	2.5	50	719
2	7.5	50	1437
3	12.5	50	2156
4	17.5	50	2874
5	22.5	50	3593
6	27.5	50	4311
7	32.5	50	5030

$$G_{total} = 10\frac{35}{2}5030 = 880{,}250 \quad \text{cubic feet}$$

Application to our "ideal" forest

- How much will we cut each 5-year period?

	Mid point		
Age Class	of age class	Acres	Volume
1	2.5	50	719
2	7.5	50	1437
3	12.5	50	2156
4	17.5	50	2874
5	22.5	50	3593
6	27.5	50	4311
7	32.5	50	5030

$$G_R = 50 \times 5030 = 251{,}500 \quad \text{cubic feet}$$

Application to our "ideal" forest

- How much will we cut over 35 years (one rotation)?

	Mid point		
Age Class	of age class	Acres	Volume
1	2.5	50	719
2	7.5	50	1437
3	12.5	50	2156
4	17.5	50	2874
5	22.5	50	3593
6	27.5	50	4311
7	32.5	50	5030

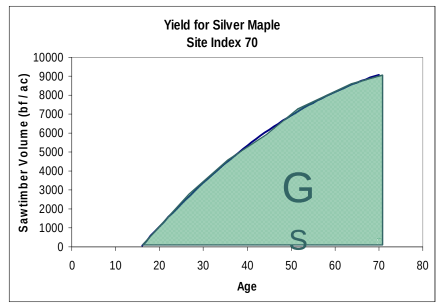
$$G_R = 7 \times 251{,}500 = 1{,}760{,}500 \quad \text{cubic feet}$$

This is 2x the total growing stock on the forest!

Useful formulas for forest regulation and allowable cut

- Growing stock (G_S)
  - This is the sum of all standing volume in the forest
  - The integral of the yield equation
  - The sum of all yields at each age

$$Y = -4588 + 318.7A - 1.765A^2$$



Determining growing stock by integration…

1. Yield equation $Y = -4588 + 318.7A - 1.765A^2$

2. Integral of yield equation

$$G_S = \int_0^R -4588A + \frac{318.7}{2}A^{1+1} - \frac{1.765}{3}A^{2+1} + c$$

3. Solve integral for values 0 to rotation age

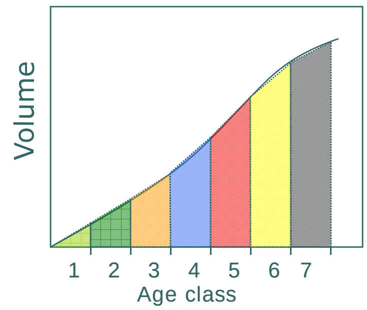
$$G_S = \int_0^R -4588A + 159.35A^2 - 0.58833A^3 + c\Big|_0^{70} =$$

= [-4588(70) + 159.35(70)^2 – 0.58833(70)^3 +c] –[-4588(0) + 159.35(70)^2 – 0.58833(70)^3 +c]

= 257,856 board feet on R acres, or 257,856 bd ft on 70 acres

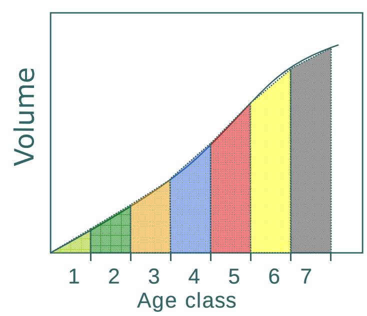
Determining growing stock using a yield table

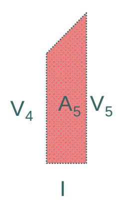
- What if we don't have a yield equation?
- We can approximate the area under the curve with age class data



Determining growing stock using a yield table

- What if we don't have a yield equation?
- We can approximate the area under the curve with age class data





$$A_5 = I\frac{V_4 + V_5}{2}$$

So, a general formula for summing age classes is..

$$G_S = \left[\frac{V_0 + V_1}{2}\right]I + \left[\frac{V_1 + V_2}{2}\right]I + \left[\frac{V_2 + V_3}{2}\right]I + \ldots\left[\frac{V_{n-1} + V_n}{2}\right]I$$

which simplifies to...

$$G_S = I\left[\frac{V_0}{2} + V_1 + V_2 + V_3 + \ldots V_{n-1} + \frac{V_n}{2}\right]$$

and since $V_0 = 0$, simplifies further to...

$$G_S = I\left[V_1 + V_2 + V_3 + \ldots V_{n-1} + \frac{V_n}{2}\right]$$

Defining variables for yield table summation…

$$G_S = I\left[V_1 + V_2 + V_3 + \ldots V_{n-1} + \frac{V_n}{2}\right]$$

G_s = growing stock volume on S acres for fully regulated or normal forest

s = number of years in rotation age and acres in forest

I = increment between age classes in yield table

V_n = yield table volume for age n

Applying this to our problem

- Assume the yield table for SI = 70 silver maple
- Applying this to a 70 year rotation, yield table summation calculates G_S…

Age	Volume (Bd. ft. / ac)
0	0
10	0
20	1080
30	3385
40	5336
50	6935
60	8180
70	9073

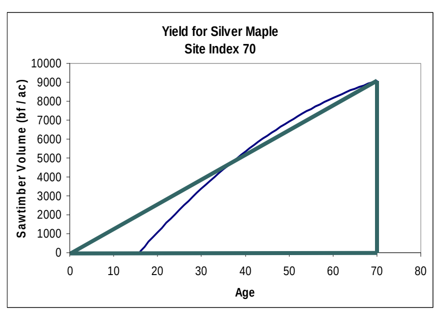
$$G_S = 10\left[V_{10} + V_{20} + V_{30} + V_{40} + V_{50} + V_{60} + \frac{V_{70}}{2}\right]$$

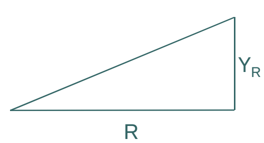
$$G_S = 10\left[0 + 1080 + 3385 + 5336 + 6935 + 8180 + \frac{9073}{2}\right] = 294{,}525$$

board feet on 70 acres

Growing stock by method of “glorious simplicity”

- We can assume that yield curve is approximated by a triangle
- Then, formula for G_S is very simple…

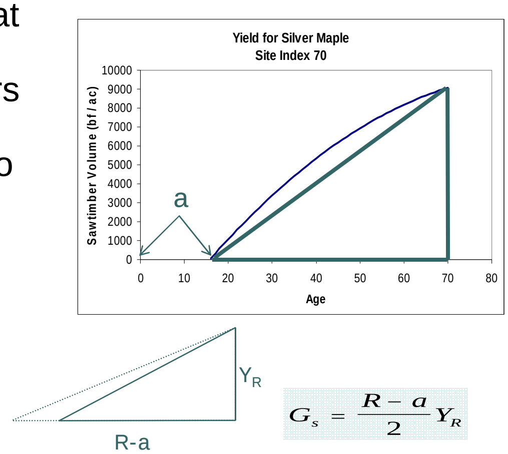




$$G_s = \frac{R}{2} Y_R$$

A better form of the method of "glorious simplicity"

- We can see that there is some number of years (a) before volume starts to accumulate
- We can adjust the base of our triangle by subtracting a from R



Applying the method of "glorious simplicity"…

Without adjustment

$$G_s = \frac{R}{2} Y_R = \frac{70}{2} 9073 = 317{,}555$$ board feet on 70 acres

With adjustment (a=15 yrs)

$$G_s = \frac{R-a}{2} Y_R = \frac{55}{2} 9073 = 249{,}508$$ board feet on 70 acres

Comparing all methods of determining growing stock

- By integration
  - G_s=257,856 bf on 70 acres
- By yield table summation
  - G_s=294,525 bf on 70 acres
- Glorious simplicity (unadjusted)
  - G_s=317,555 bf on 70 acres
- Glorious simplicity (adjusted)
  - G_s=249,508 bf on 70 acres

Next lecture

Allowable cut determination